

Consider the polar equation $r = 3 - 2\cos\theta$.

$$1 < \left| \frac{3}{2} \right| < 2$$

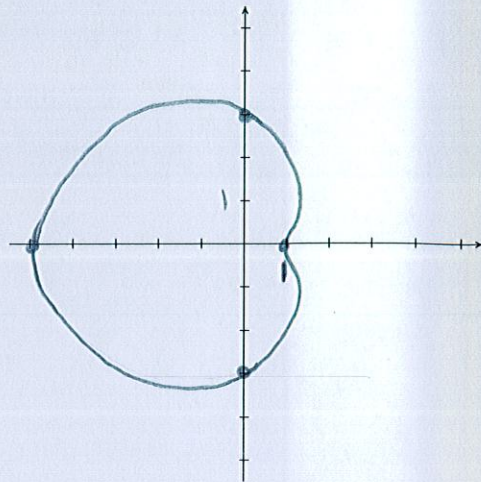
SCORE: ____ / 20 PTS

- [a] What is the shape of the graph of the equation?

LIMACON WITH DIMPLE

- [b] Sketch the graph using the shortcut process shown in lecture / PowerPoint.
Label the scale on your axes clearly.

| θ | r |
|------------------|-----|
| 0 | 1 |
| $\frac{\pi}{2}$ | 3 |
| π | 5 |
| $\frac{3\pi}{2}$ | 3 |



Find the rectangular equation of the ellipse with foci $(-5, 7)$ and $(-5, -1)$ and minor axis of length 8.

SCORE: ____ / 15 PTS

CENTER $(-5, \frac{7+(-1)}{2}) = (-5, 3)$



$$\frac{(x+5)^2}{16} + \frac{(y-3)^2}{32} = 1$$

$$4^2 + 4^2 = 32$$

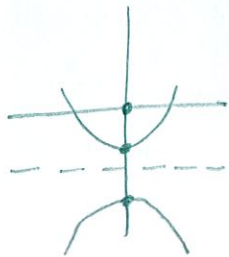
A hyperbola has a focus at the pole and vertices with rectangular co-ordinates $(0, -3)$ and $(0, -7)$.

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- [a] Find polar co-ordinates for the vertices, using positive values of r and θ .

$$(3, \frac{3\pi}{2}) \quad (7, \frac{3\pi}{2})$$

- [b] Find the polar equation of the hyperbola. Do NOT find the rectangular equation.



$$r = \frac{ep}{1 - e \sin \theta}$$

$$\text{USE } (-7, \frac{\pi}{2}) \text{ AND } (3, \frac{3\pi}{2})$$

$$-7 = \frac{ep}{1 - e}$$

$$3 = \frac{ep}{1 + e}$$

$$ep = -7 + 7e$$

$$ep = 3 + 3e$$

$$-7 + 7e = 3 + 3e$$

$$e = \frac{5}{2}$$

$$ep = 3 + 3(\frac{5}{2}) = \frac{21}{2}$$

$$r = \frac{\frac{21}{2}}{1 - \frac{5}{2} \sin \theta}$$

$$r = \frac{21}{2 - 5 \sin \theta}$$

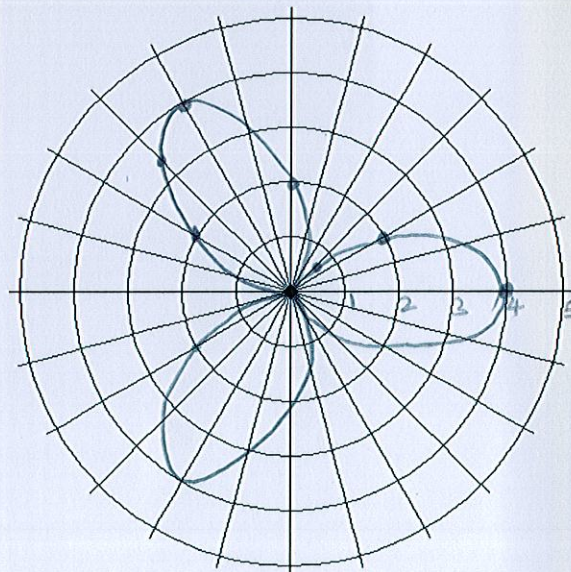
$$(-r, \theta), (-r, -\theta) \text{ and } (r, \pi + \theta)$$

- NOTE: Run as FEW tests as needed to prove your conclusions are correct.**

POLAR AXIS: $(r, -\theta)$ $r = 2 + 2\cos 3(-\theta)$
 $r = 2 + 2\cos 3\theta$

- $[0, \pi]$

- | | | | |
|--------------------|---------------|----------------------|---------------|
| $\frac{\theta}{0}$ | $\frac{r}{4}$ | $\frac{\theta}{\pi}$ | $\frac{r}{0}$ |
| $\frac{\pi}{6}$ | 2 | $\frac{5\pi}{6}$ | 2 |
| $\frac{\pi}{4}$ | ≈ 0.6 | $\frac{3\pi}{4}$ | ≈ 3.4 |
| $\frac{\pi}{3}$ | 0 | $\frac{2\pi}{3}$ | 4 |
| $\frac{\pi}{2}$ | 2 | | |



Convert the polar equation $r = 1 - \cot \theta$ to rectangular and simplify.

SCORE: _____ / 15 PTS

$$\sqrt{x^2 + y^2} = 1 - \frac{x}{y}$$

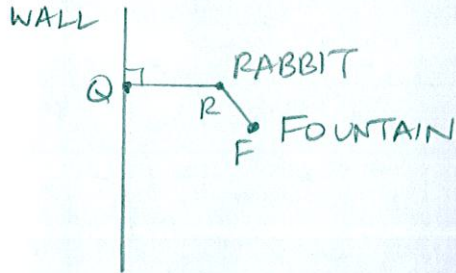
$$y\sqrt{x^2 + y^2} = y - x$$

$$y^2(x^2 + y^2) = (y - x)^2$$

A drinking fountain is 6 feet from the wall of a school building. A rabbit is running on the school grounds, so that it is always twice as far from the wall as it is from the fountain. What is the shape of the rabbit's path?

SCORE: _____ / 10 PTS

Draw a diagram and write algebraic equations involving distances to justify your answer.



$$RQ = 2RF$$

$$\frac{1}{2} = \frac{RF}{RQ} = e$$

$$\frac{1}{2} < 1 \rightarrow \text{ELLIPSE}$$

Consider the conic with rectangular equation $2x^2 + 8x - 9y^2 + 54y - 1 = 0$.

SCORE: ____ / 20 PTS

[a] What is the shape of the graph of the equation ?

HYPERBOLA

[b] Find the co-ordinates of the focus/foci.

$$2(x^2 + 4x + 4) - 9(y^2 - 6y + 9) = 1 + 8 - 81$$

$$2(x+2)^2 - 9(y-3)^2 = -72$$

$$\frac{(y-3)^2}{8} - \frac{(x+2)^2}{36} = 1$$

$$c^2 = 8 + 36 = 44$$

$$c = 2\sqrt{11}$$



FOCI

$$(-2, 3 \pm 2\sqrt{11})$$

Consider the polar equation $r = \frac{60}{7+3\cos\theta} = \frac{\frac{60}{7}}{1+\frac{3}{7}\cos\theta}$

SCORE: ____ / 20 PTS

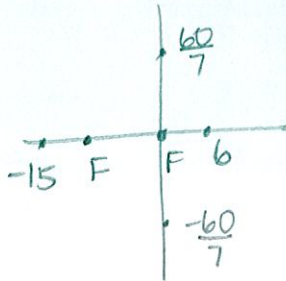
- [a] What is the shape of the graph of the equation?

$$e = \frac{3}{7} < 1 \rightarrow \text{ELLIPSE}$$

- [b] Find the rectangular coordinates of the endpoints of all latera recta. Do NOT convert the equation to rectangular.

| θ | r |
|------------------|----------------|
| 0 | 6 |
| $\frac{\pi}{2}$ | $\frac{60}{7}$ |
| π | 15 |
| $\frac{3\pi}{2}$ | $\frac{60}{7}$ |

LR $\left[\begin{array}{c} 0 \\ \frac{\pi}{2} \\ \pi \\ \frac{3\pi}{2} \end{array} \right] \left[\begin{array}{c} 6 \\ \frac{60}{7} \\ 15 \\ \frac{60}{7} \end{array} \right] \checkmark$



OTHER FOCUS $(-15+6, 0) = (-9, 0)$

ENDPOINTS OF LR

$$(0, \pm \frac{60}{7}) \quad (-9, \pm \frac{60}{7})$$